SIGNAL PROCESSING IV: Theories and Applications
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NEW DIGITAL ALGORITHMS FOR ON-LINE DETERMINATION OF SYMMETRICAL COMPONENTS IN POWER SYSTEMS

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Using the Fourier method, the Kalman theory and the Walsh functions, non-recursive algorithms for the on-line determination of the basic waveforms of the symmetrical components in three-phase power systems are introduced, investigated and compared. The circular rotating phasors of the symmetrical components are calculated from the orthogonal components instead of the three original signals. The numerical operations required for the on-line calculation can be reduced by one third. The developed algorithms combine the suppression of higher frequencies and the separation of the symmetrical positive- and negative-sequence components.

1. Introduction

The method of symmetrical components (0,1,2-components) is one of the basic tools for the analysis of three-phase power systems. The components, especially the positve- and negative-sequence one, describe the rotating system. During unbalanced disturbances they significantly change their values. The fast estimation of the symmetrical components of voltages and currents from measured signals with high noise level therefore can be efficiently used for control and protection tasksin electrical power systems.
A lot of research has been done in recent years on digital control and protection. Various algorithms have been suggested to estimate the fundamental waveforms of noisy signals. The most widely used Fourier technique [1,2] provides a good accuracy for measured signals with harmonic components. They are suitable for the estimation under steady-state conditions. In case of transients, a non-recursive version of the Kalman filters is proposed, because the fault-induced noise is random in natur [3]. The methods based on the Walsh functions can be adapted very easily to simple computer systems, because they do not need any complicated calculation support.
The phasors of the symmetrical components can be calculated either directly from the sampled three-phase signals f_R, f_S, f_T, or after a coordinate transformation, from the complex space phasor $f_\alpha + j f_\beta$ [4].
Non-recursive algorithms applied to this phasor to calculate the symmetrical components are introduced in this paper and the transfer functions obtained from the developed filter algorithms for the complex input and output signals are investigated and compared.

For the applications in protection and control equipment in electrical power systems fast methods with special filter properties in the transient state are required. For the design of the filters a program has been elaborated, which allows to develope algorithms with a special frequency behaviour. The paper presents an application for three-phase signals with thyristor converters.
The filter design includes also the considera-tion of various sampling windows and sampling frequencies. Together with this a filter design is possibly depending on the relevant applica-tions for measurement, protection or control in three-phase systems.

2. Estimation of the basic components of real signals

2.1 Fourier algorithm

The waveforms of voltages and currents in electrical power systems usually are a combination of the fundamental waveform, harmonics and random noise due to faults and other disturbances.
The fundamental waveform of the signal may be described as:

$$g(t) = C_c \cos(\omega t) + C_s \sin(\omega t) \tag{1}$$

$$g(t) = \mathrm{Re}\{\underline{G}\, e^{j(\omega t+\varphi)}\} = \mathrm{Re}\{\underline{g}(t)\}$$

The complex phasor is given by

$$\underline{g}(t) = C_c - j\, C_s \tag{2}$$

The coefficients C_c and C_s are calculated from the sampled values by the method of the least squares [2].
In this paper the sampled values are denoted by f_n, the calculated values by g_n.

$$C_c = \frac{1}{AD-B^2} \sum_{n=0}^{N-1} f_{N-1-n}[A\cos(N-n)\omega T - B\sin(N-n)\omega T] \tag{3a}$$

$$C_s = \frac{1}{AD-B^2} \sum_{n=0}^{N-1} f_{N-1-n}[D\sin(N-n)\omega T - B\cos(N-n)\omega T] \tag{3b}$$

with

$$A = \sum_{n=0}^{N-1} \sin^2(n\omega T) \quad ; \; D = \sum_{n=0}^{N-1} \cos^2(n\omega T)$$

$$B = \sum_{n=0}^{N-1} \sin(n\omega T)\cos(n\omega T)$$

T = sampling cycle, N = sampling window, ω = fundamental frequency.

For the time $t_m = t_0 + m \cdot T$ the filter output signal is given as:

$$g(t_m) = g_m = C_c\cos(m\omega T) + C_s\sin(m\omega T) \tag{4}$$

where t_0 means the beginning of the sampling window.
In order to develope equation (4) to the form of a non-recursive filter [5,6]

$$g_m = \sum_{n=0}^{N-1} f_{N-1-n}\, b_n \tag{5}$$

together with (3) the output signals is:

$$g_m = \sum_{n=0}^{N-1} f_{N-1-n}\, [b_{cn}\cos(m\omega T) + b_{sn}\sin(m\omega T)] \tag{6}$$

with

$$b_{cn} = \frac{1}{AD-B^2}[A\cos(N-n)\omega T - B\sin(N-n)\omega T]$$

$$b_{sn} = \frac{1}{AD-B^2}[D\sin(N-n)\omega T - B\cos(N-n)\omega T] \tag{7}$$

2.2 Kalman algorithms

The two-state Kalman filter can be used for the estimation of C_s and C_c in (1) [3]. For sampled values f(0) to f(N-1) the initial values $C_c(1)$ and $C_s(1)$ are calculated from the first two samples. After that the estimated values are improved (N-2) times. In this way a N-order non-recursive algorithm for the estimation of the fundamental wave form can be derived step by step.
For the real time implementation, the coefficients b_{cn} and b_{sn} are calculated off-line in order to store the constants for the digital convolution to the signal processing unit.

2.3 Walsh algorithms

In this case the fundamental wave form of the signals is approximated by the orthogonal rectangular Walsh functions [7].
The coefficients of the Walsh algorithms for a sampling window of one cycle of the basic component are

$$b_{cn} = \frac{2\,k_w}{N} \begin{cases} 1 & \text{for } 0 \le \omega t < \frac{\pi}{2} \;;\; \frac{3}{2}\pi \le \omega t < 2\pi \\ -1 & \text{for } \pi \le \omega t < \frac{3}{2}\pi \end{cases}$$

$$b_{sn} = \frac{2\,k_w}{N} \begin{cases} 1 & \text{for } 0 \le \omega t < \pi \\ -1 & \text{for } \pi \le \omega t < 2\pi \end{cases} \tag{8}$$

where $k_w \approx 0.774$.
As the coefficients differ only in sign, the realisation of the convolution is very simple, as the multiplicator is only +1 or -1.

3. Separation of the symmetrical components

Three-phase systems advantageously are described with complex phasors. In case of a symmetrical RST system the three sine-wave signals have the same amplitude and a difference of $2\pi/3$ in their phase angle. In this case the shape of the complex phasor describes a positiv-sequence circle in fig. 1a. The orthogonal axes are calculated from the RST axes by the coordinate transformation (9):

$$\begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} f_R \\ f_S \\ f_T \end{bmatrix} \tag{9}$$

The circle in fig. 1a can be deformed for example by the loss of the signal f_R to an elliptic shape in fig. 1b. This elliptic field incorporates a positiv-sequence and a negative-sequence circular component. The positiv-sequence part, the so called positive-sequence symmetrical component is important for various applications in electrical power systems. Furthermore harmonic oszillations to the basic frequency can influence the signals as shown in fig. 1c for the 5th and the 11th harmonic. The proposed filter-algorithms now suppress the harmonic oscillations and also eliminate the positiv-sequence circular part.

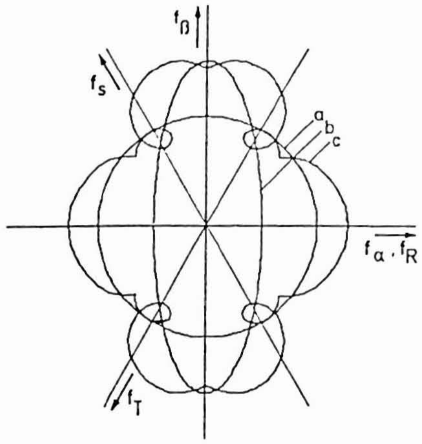

Fig. 1 Complex space phasor of the three-phase signals
a) symmetrical RST-system,
b) Unsymmetry
c) Unsymmetry and additional 5th and 11th harmonics

The complex symmetrical component

$$g_1(t) = g_{1r}(t) + jg_{1i}(t) \tag{10}$$

is the output signal to the complex input $g_\alpha(t) + jg_\beta(t)$, while the real part $g_{1r}(t)$ and the imaginary part g_{1i} are the result of the complex convolution in (11).

$$g_1 = \frac{1}{\sqrt{6}} \sum_{n=0}^{N-1} [(f_{\alpha,N-1-n} b_{cn} + f_{\beta,N-1-n} b_{sn}) + $$

$$+ j(f_{\beta,N-1-n} b_{cn} + f_{\alpha,N-1-n} b_{sn})] \tag{11}$$

The coefficient b_c and b_s are calculated by the Fourier, Kalman or Walsh methods as proposed in Kap. 2.

4. Frequency behaviour of the Algorithms

The magnitude transfer functions obtained from the developed algorithms have been investigated. The transfer properties between the complex space phasor $f_\alpha + jf_\beta$ and the complex phasor g_1 at the output are compared. The sampling frequency is set to 1000 Hz, the sampling window covers one period of the basic waveform. The results are shown in fig. 2. As the magnitude of the transfer function is zero for $f = -50$ Hz, it indicates the negative-sequence component to be totally suppressed. The Fourier algorithm in fig.2a offers the best suppression of harmonic frequencies, while the Kalman algorithm fig.2b improves the rejection of the non-harmonic random noise. Compared to that, the behaviour of the Walsh algorithm fig.2c is less satisfactory for the applications in

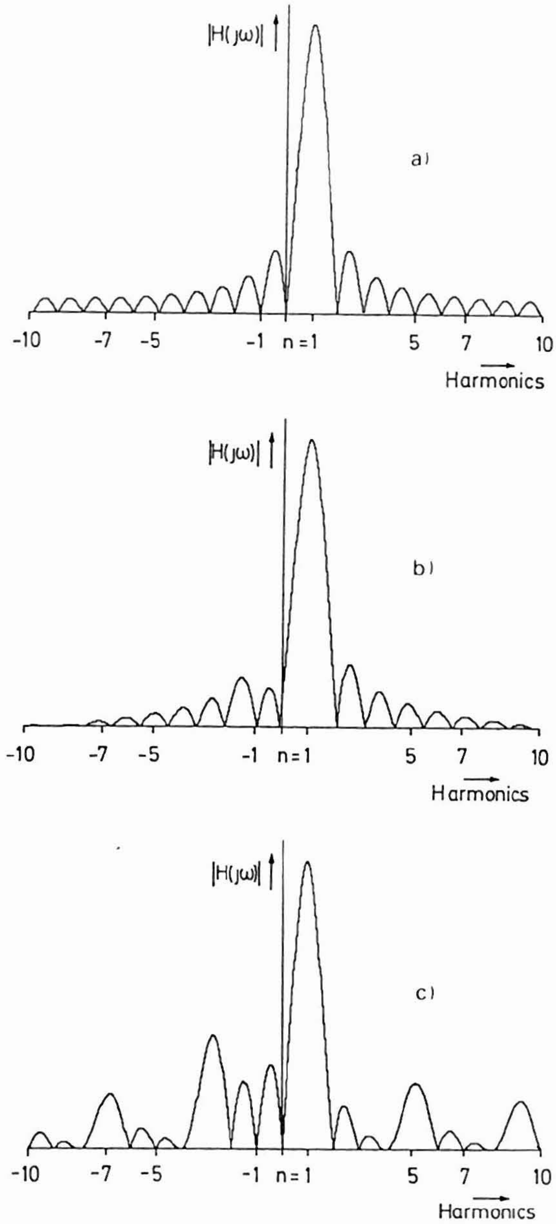

Fig.2 Transfer functions of the complex filteralgorithms
a) Fourier, b) Kalman, c) Walsh algorithms. Sampling frequency f=1000 Hz, filter order N=20

electrical power systems because of the insufficient suppression of odd harmonics.

5. Algorithms with special properties

Some applications in protection and control in electrical power systems require fast algorithms with special filter properties in frequency domain. A filter-design program for these purposes has been elaborated, which also allows to involve interpolative constraints [8]. According to this, the filter response can be predetermined for special frequencies.

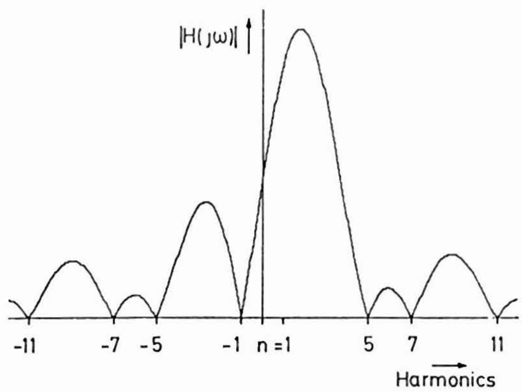

Fig. 3 Transfer function of the complex filter-algorithm. Design with interpolative Constraints [8]. Sampling frequency f=1200 Hz, Filter order N=8.

Fig.3 shows the transfer function of a developed algorithm to calculate the positive-sequence components. As constraints the frequency response is zero at the 5th, 7th and 11th harmonic. This behaviour can be used in three-phase circuits with thyristor converters, where these harmonics have significant values.

6. Conclusions

Using the new algorithms to calculate the complex phasors of the symmetrical component g_1 from the complex space phasors $f_\alpha + jf_\beta$, the numerical operations can be reduced by one third compared with [2]. The most widely used algorithms for on-line estimation of basic components of signals based on the Fourier technique are exact, if the measured signals contain only harmonic components. The non-recursive algorithm, based on the Kalman filter theory is more suitable in cases of random noise signals. The methods based on the Walsh function are very simple for computer implementation, because they do not need multiplications. However, the filter properties of these algorithms are not convenient, especially for the odd harmonics. The new approach enables also to develope complex algorithms with special filter properties in frequency domain.

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References

[1] Phadke, A.G.; Thorp, J.S. and Adamiak, M.G.: A New Measurement Technique for Tracking Voltage Phasors, Local Frequencies and Rate of Change of Frequency, IEEE Trans., PAS-102, (1983) pp.1025-1038

[2] Hosemann, G.; Łobos, T.: Ermittlung der symmetrischen Komponenten durch Abtastalgorithmen. Archiv f. Elektrotechnik 68 (1985) pp. 1-16.

[3] Łobos, T: Non-recursive Methods for On-Line estimation of the Fundamental Waveforms of Signals Using Kalman Filter Theory, 3rd European Signal Processing Conference, (The Hague 1986) pp.993-996

[4] DIN 13321: Komponenten in Drehstromnetzen. Begriffe, Größen, Formelzeichen. Berlin: Beuth-Vertrieb 1980.

[5] Łobos, T, Schneider, E. and Eichhorn, K.-F.: Digital Methods for On-Line Estimation of the Basic Components of Signals in Electrical Power Systems, 9th Power Systems Computation Conference, (Cascais, Portugal 1987) pp. 739-745.

[6] Hosemann, G., Schneider, E. and Lobos, T.: Echtzeitberechnung der modalen Drehstromkomponenten nach plötzlichen Zustandsänderungen, Archiv f. Elektrotechnik 70 (1987) pp. 237-253.

[7] Hölzler, E.; Holzwarth, H.: Pulstechnik. Band I. Springer Verlag Berlin 1975, pp.46-48.

[8] Steffen, P.: Fourier-Approximation with interpolative Constraints. AEÜ Archiv für Elektrotechnik und Übertragungstechnik. Electronics and Communication. 1984, Bd. 38 pp.363-367.